Prove that $g(x)=\frac{1}{2}\ln\frac{1+x}{1-x}$ is the inverse of $f(x)=\tanh x$ by simplifying $g(f(x))$. SCORE: _____ / 5 PTS

NOTE: You may use any identities proven in part [1] of the Hyperbolic Functions Supplement WITHOUT proving them here.

(1) $\frac{1}{2}\ln\frac{1+\tanh x}{1-\tanh x}$

$= \frac{1}{2}\ln\left(\frac{1+\frac{e^x-e^{-x}}{e^x+e^{-x}}}{1-\frac{e^x-e^{-x}}{e^x+e^{-x}}}\cdot\frac{e^x+e^{-x}}{e^x+e^{-x}}\right)$ (1)

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$= \frac{1}{2}\ln\frac{e^x+e^{-x}+e^x-e^{-x}}{e^x+e^{-x}-(e^x-e^{-x})}$ (1)

$= \frac{1}{2}\ln\frac{2e^x}{2e^{-x}} = \frac{1}{2}\ln e^{2x} = \frac{1}{2}\cdot 2x = x$ (1) ($\frac{1}{2}$) ($\frac{1}{2}$)

Rewrite $\operatorname{csch}\left(\frac{1}{2}\ln 5\right)$ in terms of exponential functions and simplify.

SCORE: ______ / 3 PTS

$$\frac{2}{e^{\frac{1}{2}\ln 5} - e^{-\frac{1}{2}\ln 5}} = \frac{2}{\sqrt{5} - \frac{1}{\sqrt{5}}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{5-1} = \frac{2\sqrt{5}}{4} = \frac{\sqrt{5}}{2}$$

(1) (1) (½) (½)

Write and **prove** a formula for $\sinh(x-y)$ in terms of $\sinh x$, $\sinh y$, $\cosh x$ and $\cosh y$. **SCORE: ______ / 5 PTS**

$\sinh x \cosh y - \cosh x \sinh y$ ①

$= \dfrac{e^x - e^{-x}}{2} \dfrac{e^y + e^{-y}}{2} - \dfrac{e^x + e^{-x}}{2} \dfrac{e^y - e^{-y}}{2}$ ½

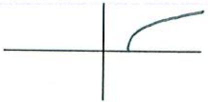
① $= \dfrac{e^{x+y} + e^{x-y} - e^{-x+y} - e^{-x-y}}{4} - \dfrac{e^{x+y} - e^{x-y} + e^{-x+y} - e^{-x-y}}{4}$ ①

½ $= \dfrac{2e^{x-y} - 2e^{-x+y}}{4}$ ½ $= \dfrac{e^{x-y} - e^{-(x-y)}}{2}$ ½ $= \sinh(x-y)$

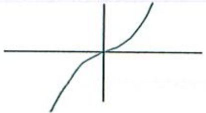
Sketch the general shape and position (including asymptotes) of the following graphs.
(Don't worry about specific $x-$ or $y-$ coordinates.)

SCORE: ______ / 3 PTS

$f(x) = \cosh^{-1} x$



$f(x) = \sinh x$



$f(x) = \tanh^{-1} x$

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There is an identity involving $\sinh x$ and $\cosh x$ called the "Pythagorean-like identity" in lecture. **SCORE: ______ / 8 PTS**

[a] Write that identity involving $\sinh x$ and $\cosh x$. **<u>You do NOT need to prove the identity.</u>**

$$\cosh^2 x - \sinh^2 x = 1$$ (1)

[b] Use the answer of [a] to find and **<u>prove</u>** an identity involving $\tanh^2 x$ that resembles a Pythagorean identity from trigonometry.
<u>NOTE: The proof shows how your answer came from the answer of [a].</u>

(1) $$\frac{\cosh^2 x - \sinh^2 x}{\cosh^2 x} = \frac{1}{\cosh^2 x} \longrightarrow 1 - \tanh^2 x = \operatorname{sech}^2 x$$ (1)

[c] Write the identity for $\cosh 2x$ that uses both $\sinh x$ and $\cosh x$ simultaneously. **<u>You do NOT need to prove the identity.</u>**

$$\cosh 2x = \cosh^2 x + \sinh^2 x$$ (1/2)

[d] Use the answers of [a] and [c] to find and **<u>prove</u>** an identity for $\cosh 2x$ that uses only $\sinh x$.

$$\cosh^2 x = \sinh^2 x + 1$$ (1/2)

$$\cosh 2x = \sinh^2 x + 1 + \sinh^2 x = 2\sinh^2 x + 1$$ (1/2) (1/2)

[e] If $\operatorname{csch} x = -\frac{5}{3}$, find $\operatorname{sech} x$ using identities.
<u>You must explicitly show the use of the identities but you do NOT need to prove the identities.</u>
<u>Do NOT use inverse hyperbolic functions nor their logarithmic formulae in your solution.</u>

$$\sinh x = \frac{1}{\operatorname{csch} x} = -\frac{3}{5}$$ (1/2)

$$\cosh^2 x = \sinh^2 x + 1$$
$$= \frac{9}{25} + 1$$ (1/2)
$$= \frac{34}{25}$$ (1/2)

(1/2) $$\cosh x = \frac{\sqrt{34}}{5} \quad (\text{SINCE } \cosh x > 0)$$ (1/2)

$$\operatorname{sech} x = \frac{1}{\cosh x}$$
$$= \frac{5}{\sqrt{34}} \cdot \frac{\sqrt{34}}{\sqrt{34}}$$
$$= \frac{5\sqrt{34}}{34}$$ (1/2)